If we subtract from the elements in the first $n-1$ columns of this determinant the corresponding elements of the last column, we shall obtain a determinant which we can easily reduce to the order $n-1$. Now subtract from the elements of the first $n-2$ lines of this new determinant the corresponding elements of the last line. We obtain after a little reduction

$$\Delta = -s_n \begin{vmatrix} -n & -n & -n & \cdots & -n \\ 0 & -n & -n & \cdots & -n \\ 0 & 0 & -n & \cdots & -n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & -n \end{vmatrix} = (-n)^{n-1}\frac{1+n}{2},$$

the determinant being of order $n-2$.

Pass now to the determinant Δ'. Putting

$$a_k = a_1 + (k-1)r \quad (k = 1, 2, 3, \cdots, n),$$

we shall have

$$\Delta' = \begin{vmatrix} a_1 & a_1 + r & a_1 + 2r & \cdots & a_1 + (n-2)r & a_1 + (n-1)r \\ a_1 + (n-1)r & a_1 & a_1 + r & \cdots & a_1 + (n-3)r & a_1 + (n-2)r \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_1 + r & a_1 + 2r & a_1 + 3r & \cdots & a_1 + (n-1)r & a_1 \end{vmatrix}.$$

The same reasoning will lead us to

$$\Delta' = (-n)^{n-1}r^{n-1}\left(a_1 + \frac{n-1}{2}r\right),$$

as it is easy to see.

II. Solution by Otto Dunkel, Washington University.

The reduction of these special circulants can be made to depend upon the known fact that the general circulant reduces to the product $f(\omega_1)f(\omega_2)\cdots f(\omega_n)$, where the ω's are the roots of $x^n - 1 = 0$ and $f(x) = a_1 + a_2x + a_3x^2 + \cdots + a_nx^{n-1}$ (see Cesàro, *Elementares Lehrbuch der algebraischen Analysis* etc., page 25). If we set $a_k = a_1 + (k-1)r$, then

$$\begin{aligned} f(\omega) &= a_1[1 + \omega + \omega^2 + \cdots + \omega^{n-1}] + r[\omega + 2\omega^2 + 3\omega^3 + \cdots + (n-1)\omega^{n-1}], \\ &= 0 + \frac{rn}{\omega - 1}, \text{ if } \omega \neq 1, \\ &= a_1n + \frac{r(n-1)n}{2} = n\left[a_1 + \frac{n-1}{2}r\right], \text{ if } \omega = \omega_1 = 1, \end{aligned}$$

whence

$$\Delta' = \frac{r^{n-1}n^n\left[a_1 + \frac{n-1}{2}r\right]}{(\omega_2 - 1)(\omega_3 - 1)\cdots(\omega_n - 1)} = (-1)^{n-1}r^{n-1}n^{n-1}\left[a_1 + \frac{n-1}{2}r\right].$$

Also solved by H. L. Olson and A. Pelletier.

2777 [1919, 268]. Proposed by W. D. Cairns, Oberlin College.

Prove that the two series

$$1 + \frac{\pi^4}{2^4\cdot 4!} + \frac{\pi^8}{2^8\cdot 8!} + \cdots$$

and

$$\frac{\pi^2}{2^2\cdot 2!} + \frac{\pi^6}{2^6\cdot 6!} + \frac{\pi^{10}}{2^{10}\cdot 10!} + \cdots$$

are equal.

Solution by H. S. Uhler, Yale University.

By substituting x, $-x$, ix, and $-ix$ for y in the absolutely convergent series

$$e^y = 1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots + \frac{y^n}{n!} + \cdots$$

it will be found at once that

$$\tfrac{1}{4}[(e^x + e^{-x}) + (e^{ix} + e^{-ix})] = 1 + \frac{x^4}{4!} + \frac{x^8}{8!} + \cdots + \frac{x^{4n}}{(4n)!} + \cdots, \tag{1}$$

$$\tfrac{1}{4}[(e^x + e^{-x}) - (e^{ix} + e^{-ix})] = \frac{x^2}{2!} + \frac{x^6}{6!} + \frac{x^{10}}{10!} + \cdots + \frac{x^{4n-2}}{(4n-2)!} + \cdots, \tag{2}$$

where $n = 1, 2, 3, \cdots$.

Again, since $e^{i\theta} = \cos\theta + i\sin\theta$ it follows that $e^{i(\pi/2)} + e^{-i(\pi/2)} = i - i = 0$.

Consequently by substituting $\pi/2$ for x in formulas (1) and (2) we see at a glance that each of the given series has the same limit $\tfrac{1}{4}(e^{\pi/2} + e^{-\pi/2})$, that is, the series are "equal."

Also solved by W. W. BEMAN, P. J. DA CUNHA, A. M. HARDING, H. L. OLSON, A. PELLETIER, S. W. REAVES, ELIJAH SWIFT, E. H. WORTHINGTON, and the Proposer.

2785 [1919, 366]. Proposed by W. H. ECHOLS, University of Virginia.

If on the sides, as bases, of any closed plane polygon, there be constructed similar triangles similarly placed, all outward or all inward, then the centroid of the vertices of these triangles coincides with the centroid of the corners of the polygon.

SOLUTION BY THE PROPOSER.

Let $Z_1, \cdots, Z_n \equiv Z_1$ be the n corners of the polygon, the Z's being complex numbers. The sides of the polygon are respectively

$$\Delta Z_r \equiv Z_{r+1} - Z_r, \qquad (r = 1, \cdots, n-1)$$

and $\Sigma\Delta Z_r = 0$, since the polygon is closed.

The n vertices of the similar triangles constructed similarly on the sides are

$$w_r = Z_r + k\Delta Z_r \cdot e^{i\alpha}, \qquad (r = 1, \cdots, n-1)$$

k being a real constant factor and α a real constant angle.

Hence,

$$\Sigma w_r = \Sigma Z_r + ke^{i\alpha}\Sigma\Delta Z_r$$

and therefore,

$$\frac{1}{n}\Sigma w_r = \frac{1}{n}\Sigma Z_r.$$

Also solved by S. W. REAVES and ELIJAH SWIFT.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

At Ohio State University, Messrs. VAN B. TEACH, V. B. CARIS and D. L. HOLL have been assistants in mathematics for the present year.

H. R. BRAHANA, of Princeton University, has been appointed instructor in mathematics at the University of Illinois for 1920–1921.

Miss MAY J. SPERRY, of Brown University, has been appointed instructor in mathematics and physics, at Knox College, Galesburg, Ill., for 1920–21.